

# Classification in Context

Adapting to changes in class and cost distribution

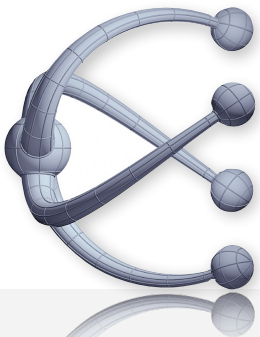
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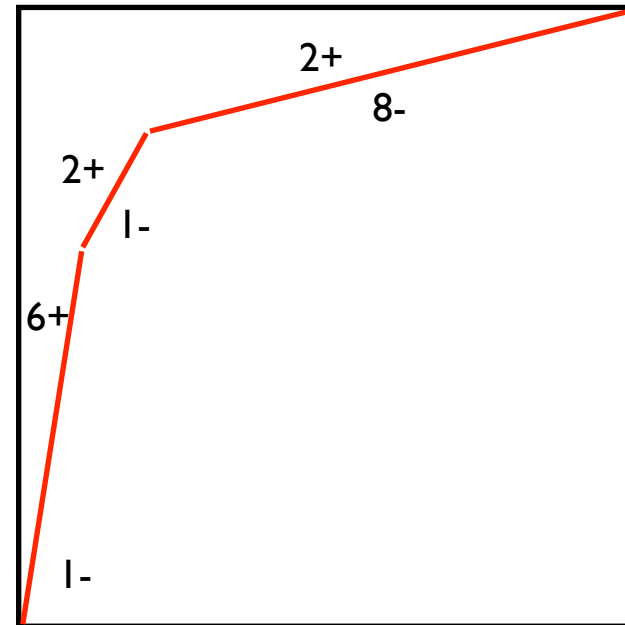
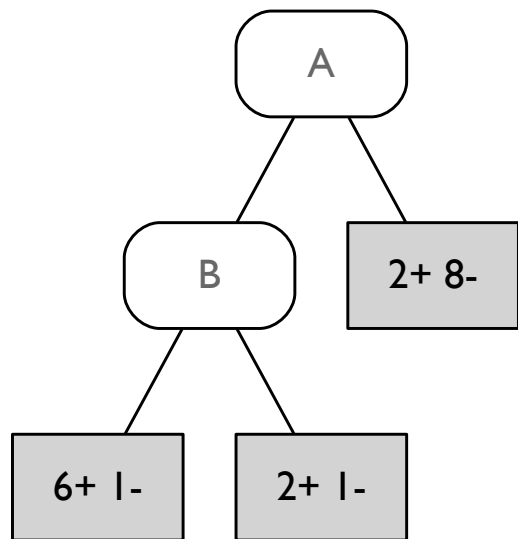


# Motivation and summary

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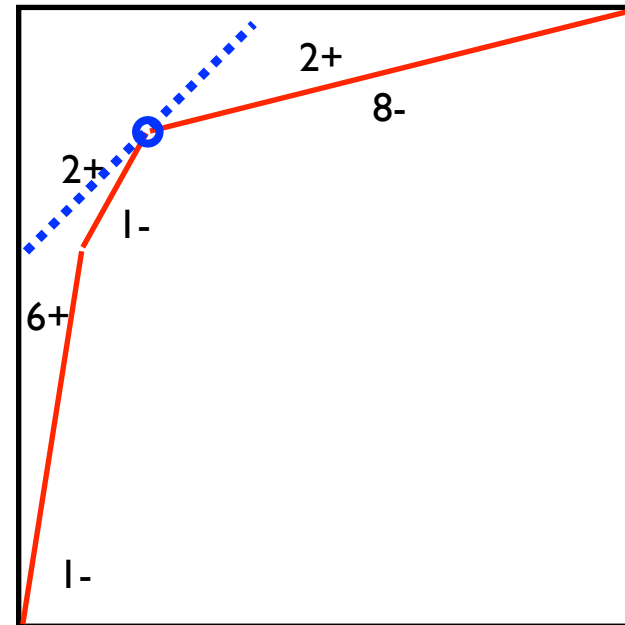
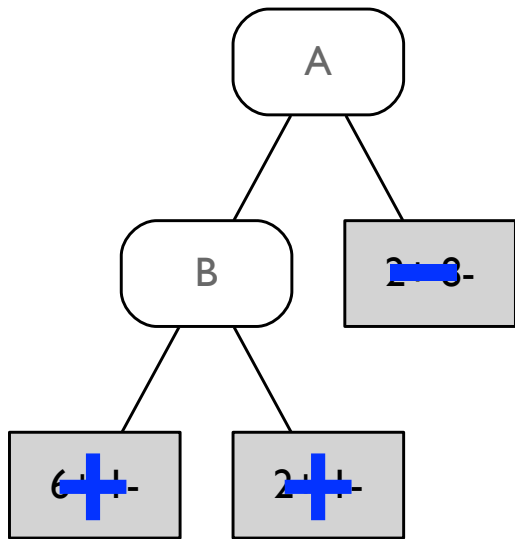
- ❖ In cost-sensitive classification, relative cost  $c$  (e.g., of misclassifying one positive relative to misclassifying one positive and one negative) provides a **context** in which loss is determined.
- ❖ This paper addresses two questions:
  - ❖ Q1: Can we reinterpret  $c$  in terms of class distributions?
    - ❖ A1: Yes, as **context change** from training to deployment. This allows us to recalibrate to such context changes.
  - ❖ Q2: What happens when we change the loss context more radically, by basing the loss on F-measure rather than accuracy?
    - ❖ A2: A lot. But we can still obtain scores calibrated to this new context.

# Decision tree and ROC curve



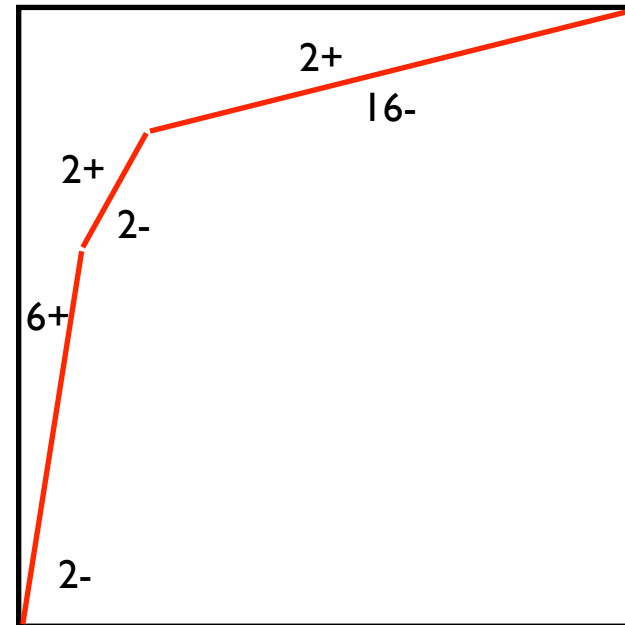
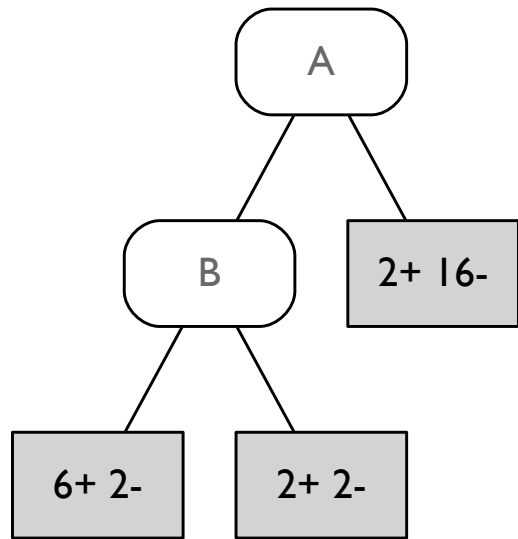
The ROC curve on the right has one segment for each leaf of the tree. The curve is always convex on the training set.

# Majority class decision rule



Labelling each leaf with its majority class achieves 80% accuracy.  
This is optimal.

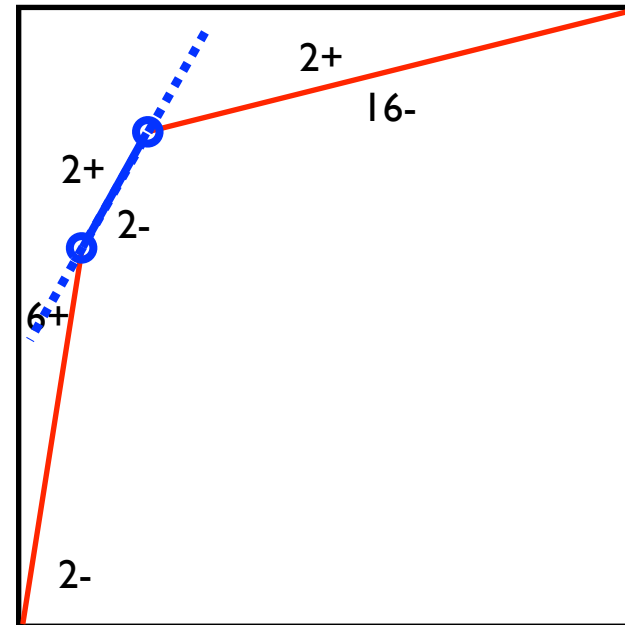
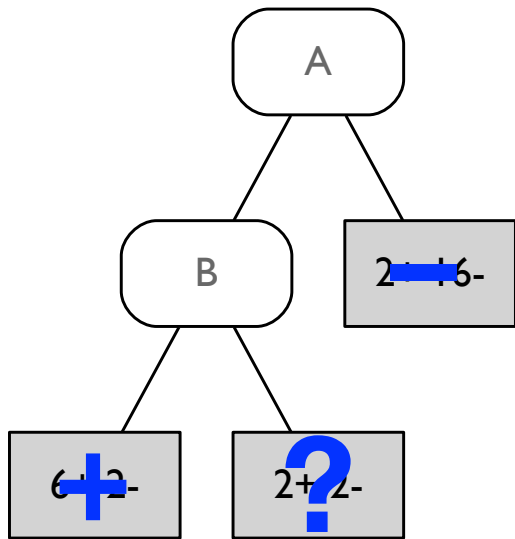
# Adapting to deployment context ( $z=1/3$ )



One way to represent the changed class distribution is by inserting another copy of each negative.

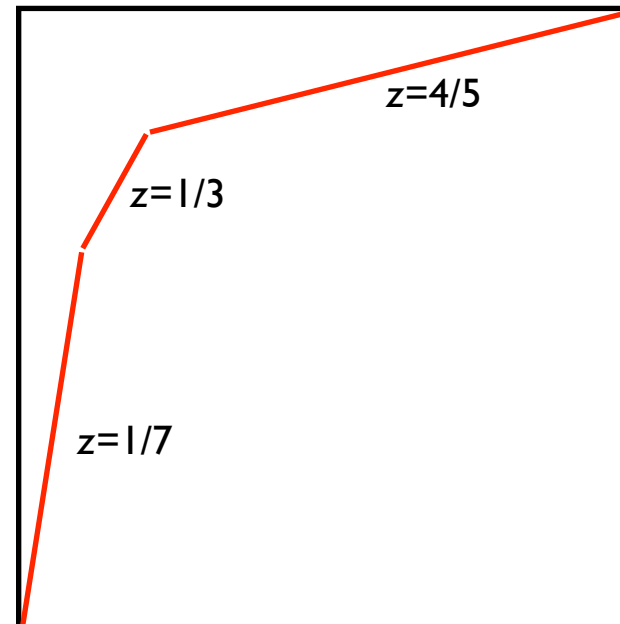
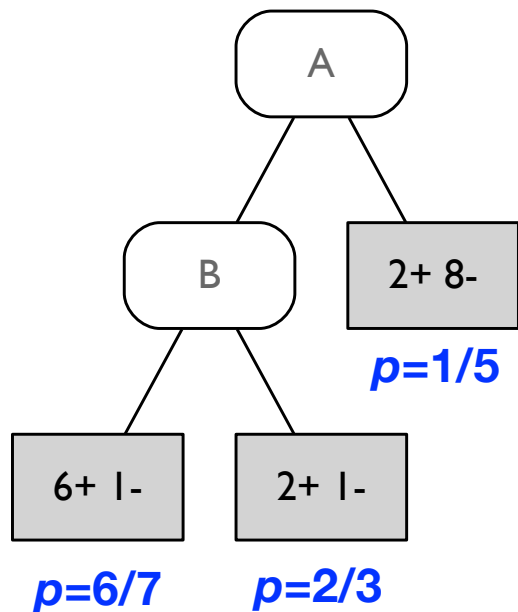
This does not change the shape of the ROC curve.

# Sitting on the fence...



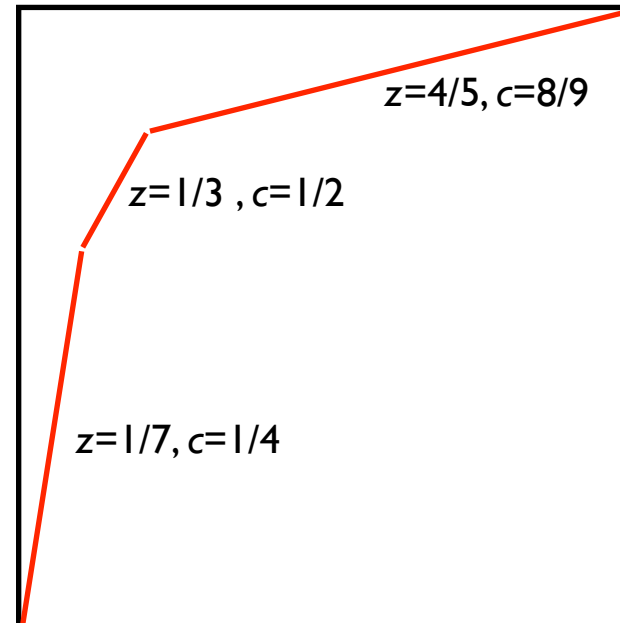
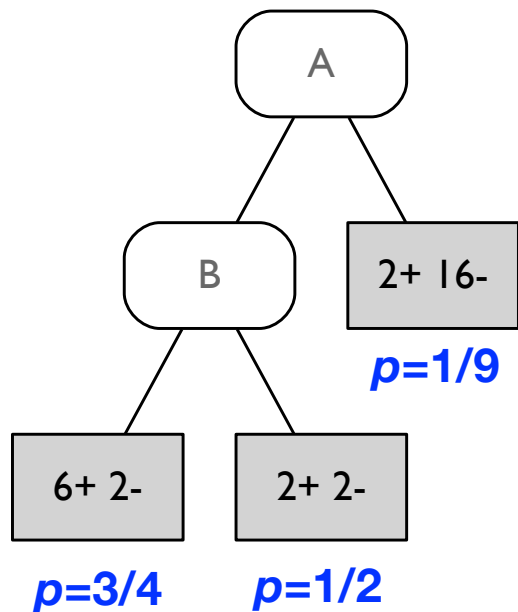
The middle leaf is now exactly on the decision boundary.  
Regardless how we label it, we misclassify 6 of the 30 instances.

# Calibration (for uniform prior)



For each leaf, determine the value of  $z$  for which it is on the decision boundary and predict positive posterior  $p=(1-z)$ .

# Calibration (for positive prior $\pi=1/3$ )

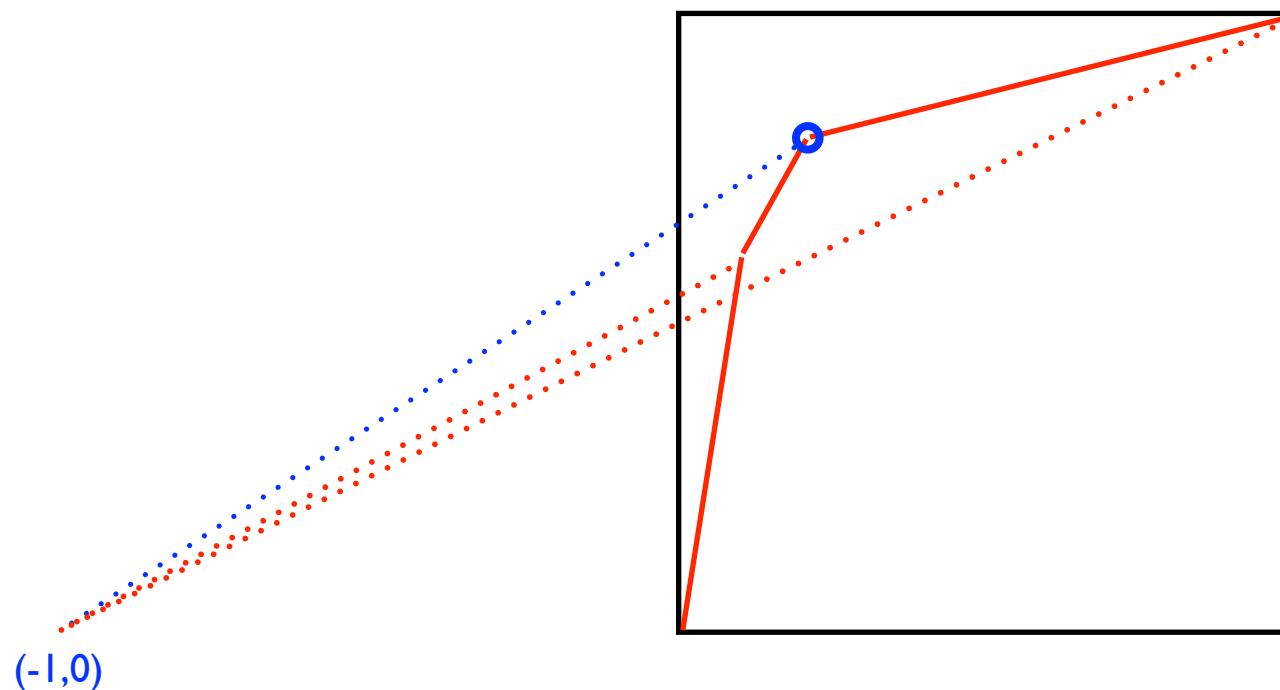


For each leaf, determine the value of  $c=(1-\pi)z/[\pi(1-z)+(1-\pi)z]$  for which it is on the decision boundary and predict positive posterior  $p=(1-c)$ .



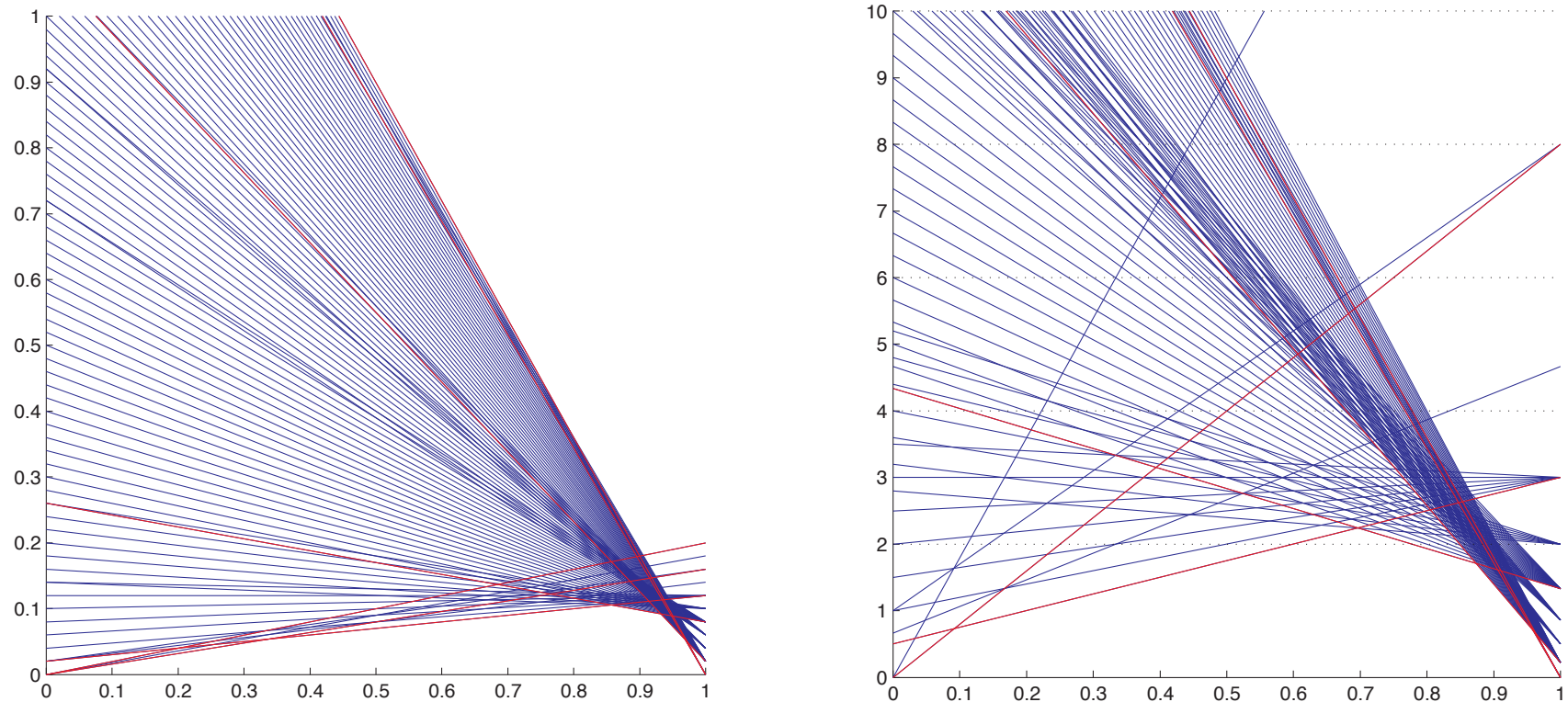
# What happens if we change loss to F-measure?

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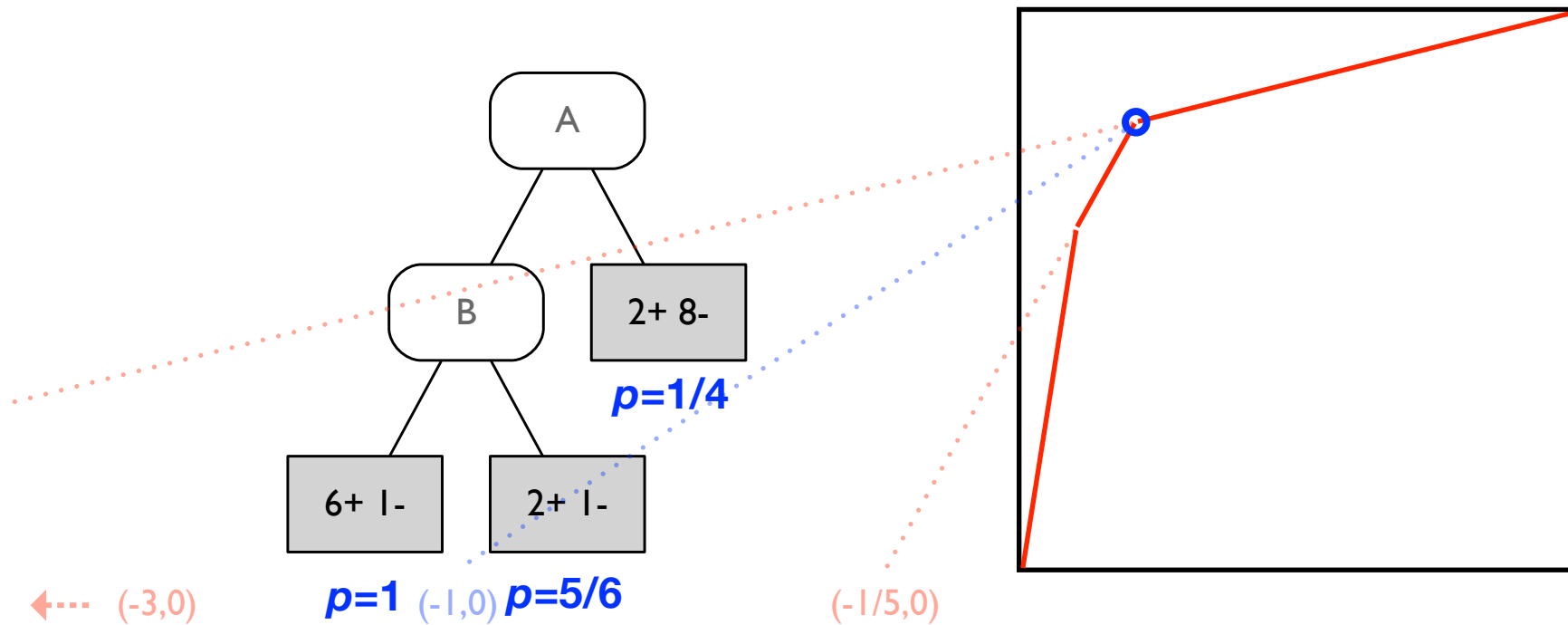
F-measure isometrics rotate around an imaginary point on the  $x$ -axis. For uniform classes this point is  $(-1, 0)$ , which allows us to find the **optimal point**. Is there a way to do this directly with scores, calibrated in some way?

# Key insight: F-cost curves



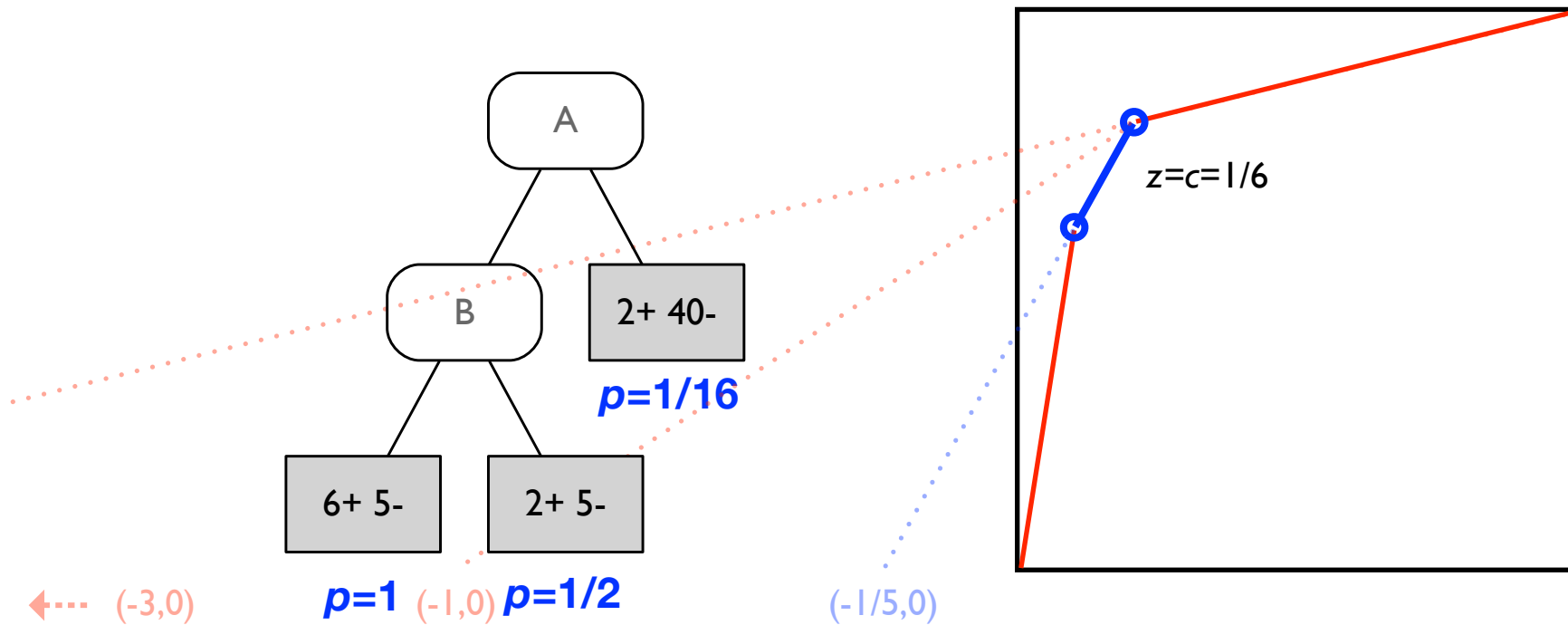
**Fig. 3. (left)** Accuracy-based cost lines. The  $x$ -axis shows  $c$  and the  $y$ -axis shows accuracy-based loss. The **red** cost lines correspond to operating points on the ROC convex hull. **(right)** Cost lines for F-measure loss. The  $y$ -axis shows  $2FQ/(1-FQ)$ . We can see that the optimal operating points are chosen for lower values of  $c$ , as expected.

# Model calibrated for F-measure (1)



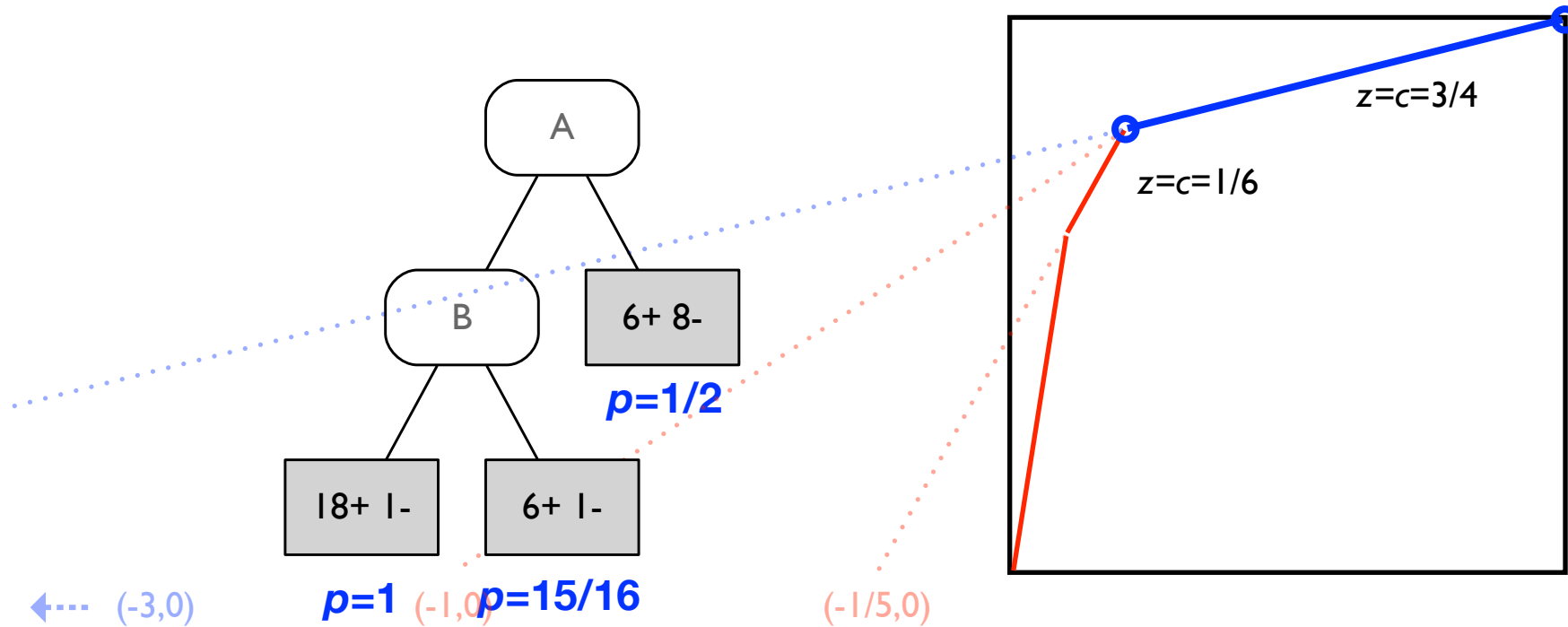
Calibrated F-measure scores for uniform classes.

# Model calibrated for F-measure (2)



Decrease the positive-to-negative ratio to 1/5.

# Model calibrated for F-measure (3)



Increase the positive-to-negative ratio to 3.

# Concluding remarks

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- ❖ One view of calibration is that a well-calibrated classifier aims to approximate the true posterior probabilities  $p(Y | X)$ .
  - ❖ Problem:  $p(Y | X)$  is not useful in determining optimal thresholds for F-measure.
- ❖ The alternative view put forward here is that classifier scores are well-calibrated for loss measure  $Q$  and context  $C$  if
  - ❖ the threshold  $1/2$  is  $Q$ -optimal in context  $C$ ;
  - ❖ more generally, the threshold  $(1-c)$  is  $Q$ -optimal in context  $C'$ , where  $c$  is the context change.
- ❖ This naturally leads to a perspective where one classifier can output multiple scores for a single instance, each calibrated for a particular loss.
  - ❖ Q: does F-calibrated score have probabilistic interpretation?