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Computational Measures of Information Gain and Reinforcement in Inference Processes

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ORGANISATION.

1. Introduction
   1.1. Framework.
   1.2. Precedents.
   1.3. Objectives.
   1.4. Methodology.
2. New Measures.
3. Applications.
4. Main Contributions.
5. Conclusions.

1. INTRODUCTION.

1.1. Framework.

Evaluation of Inference Processes

Estimate of the Value of the Result of an Inference

$$E, C \rightarrow R$$

Dimensions:

- Information Gain.
- Certainty Degree.

Utility:

- Evaluation of Conceptual Systems.
- Measurement and Development of Reasoning Systems.
- Combination of Inference Processes.

1. INTRODUCTION.

1.2. Precedents. Inference and Information.

Inference Paradox:

A) If the conclusion of an inference is not contained in the premises, it cannot be valid.

B) The conclusion cannot be contained in the premises and be at the same time novel.

?

C) Inferences cannot be at the same time ***valid*** *and* ***novel*****.**

1. INTRODUCTION.

1.2. Precedents. Inference and Information.

Carnap Probabilistic Calculus:

- if $P \models Q$ then $p(P) \leq p(Q)$

The relation between information and probability:

- $I(\mathrm{P}) = -\log p(P)$

Induction and deduction are seen as inverse processes in terms of information gain.

1. INTRODUCTION.

1.2. Precedents. Inference and Information.

(Popper & Miller 1983):

> There cannot be exclusively inductive dependence between two formulae.

(Cussens 1998):

> Corollary: Q is deductively independent from P if and only if $\neg P \models Q$.

"*Any notion of induction as a class of complement of deduction seems untenable*". (Cussens 1998).

1. INTRODUCTION.

1.2. Precedents. Inference and Effort.

Assumption: ~~**omniscience**~~. ?

- Inference must be considered potential and depth and surface information must be distinguished (Hintikka 1970).
- An agent will know an assertion which is *implicit* in its previous beliefs if it performs an inference **effort**.

*The conclusion can be contained in the premises and at the same time be novel, because it is **difficult** to make it explicit.*

How can the difficulty or effort of inference be measured?

1. INTRODUCTION.

1.2. Precedents. Inference and Confirmation.

– Classical Deduction: absolute confirmation.

– Non-monotonic, probabilistic deduction (or with uncertainty): quantitative confirmation.

– Induction / Abduction:

- Quantitative View (Carnap 1950).
- Qualitative View (Hempel 1945) (Flach 1995a).

Confirmation as effort (Quine 1953):

- *Problem: it is limited to pre-existent attributes.*

Is it possible to consider confirmation sources from different inference processes at the same time?

1. INTRODUCTION.

1.2. Precedents. Combination and Evaluation

Problem of Combination:

- Nomologic View of Induction as Deduction or Completion from general or innate laws (Hempel & Oppenheim 1965).

Evaluation:

induction
- Simplicity criteria (MDL, Rissanen 1978).
- Bayesian criteria (MLE, a priori distributions).
- Informativeness/falsifiability criteria (Popper)
- Explanation, unification or coherence criteria.

INCOMPATIBLE

deduction
- Informal Utility Criteria.
- Measures of auxiliary concepts (Hintikka 1973).

Is it possible to develop unified measures (or at least compatible) for different inference processes?

1. INTRODUCTION.

1.3. Objectives.

Development of compatible measures for evaluating the result of the inferential synthesis of concepts in terms of information gain and reinforcement.

Dimensions:

- Informativeness
- Plausibility
- ‘Consilience’
- Intensionality
- Comprehensibility / Intelligibility
- Utility

1. INTRODUCTION.

1.4. Methodology.

Separate the measures of information and confirmation.

- Modern view of <u>information</u> theory:

The ***Kolmogorov Complexity*** of an object x given y is :

$$K(x|y) = \min \{ l(p) : \phi(p,y) = x) \}$$

The absolute complexity of an object is $K_\beta(x) = K_\beta(x|\varepsilon)$.

- Measure of computational <u>effort</u>:
  - Weighting of Space and Time through *LT*.

$$LT_\phi(p_x) = l(p_x) + log_2 \mathrm{Cost}_\phi(p_x)$$

The ***Levin Complexity*** of an object x given y is:

$$Kt(x|y) = \min \{ LT_\phi(p) : \phi(p,y) = x \}$$

1. INTRODUCTION.

1.4. Methodology.

- View of <u>inference</u> from a strictly <u>computational</u> point of view .
- Quantitative view of <u>confirmation</u> but not probabilistic (as reinforcement)
- Some dimensions depend on <u>detailed measurements</u> for parts of any concept / theory, and not a joint value.
- The conviction that <u>reasoning systems</u> can also <u>be evaluated</u> by measures exclusively derived formally and computationally.

2. NEW MEASURES.

2.1. Computational Information Gain.

The ***time-independent information gain*** of an object x wrt. an object y is defined as

$$V(x \mid y) = K(x \mid y) \;/\; K(\mathrm{x})$$

The ***computational information gain*** *(space-temporal)* of an object x wrt. an object y is defined as:

$$G(x \mid y) = Kt(x \mid y) \;/\; Kt(\mathrm{x})$$

- Their properties of limits and robustness are studied.
- They are compared with other gain measures. (Quinlan 1993)

2. NEW MEASURES.

2.1. Computational Information Gain.

- If $G \approx 1$, it is due to:
  - a computational effort, or
  - independent information.

How can both cases be distinguished?

The ***real gain of information*** of an object x wrt an object y is:

$$TG(x \mid y) = [Kt(x \mid y) - K(x \mid y)] \,/\, Kt(x)$$

V(x | y)	G(x | y)	TG(x | y)	V(x | y)/G(x | y)	Meaning
1	1	0	1	x is not implicit nor explicit in y
0	1	1	0	x is deeply implicit in y
1	≅ 0	-	-	Impossible
0	≅ 0	≅ 0	0	x is explicit in y

2. NEW MEASURES.

2.2. Gain and Inference Processes.

INDUCTION: if x is the theory and y is the evidence:

- Minimum: $G(x \mid y) = \log l(x) / (l(x) + \log(l(x)) \approx 0$.
  The theory is evident from the data.
- Maximum: $G(x \mid y) = 1$.
  The theory is surprising wrt. the data.

Oblivion Criterion. Given a plausibility criterion $PC(h \mid d)$, its memory politics can be ruled by:

$$OC(h \mid d) = G(h \mid d) \cdot PC(h \mid d)$$

2. NEW MEASURES.

2.2. Gain and Inference Processes.

DEDUCTION: if x is the conclusion and y are the premises:

- Minimum: $G(x \mid y) = \log l(x) / (l(x) + \log(l(x)) \approx 0$.
  The conclusion is evident from the premises.
- Maximum: $G(x \mid y) = 1$.
  The conclusion is surprising wrt. the premises.

Several measures of optimality of axiomatic systems are established, which weight the effort of derivation of new facts with the size of the system (number of rules made explicit).

2. NEW MEASURES.

2.2. Properties of gain measures.

The gain measures introduced:

- Constitute a descriptional mathematisation of Popper’s view of informativeness for induction.
- Generalise Hintikka’s view of deep and surface information.
- Subsume other measures of information gain for decision trees (Quinlan 1986, 1990).
- Clarify and overcomes the inference paradox.

2. NEW MEASURES.

2.3. Measure of Constructive Reinforcement.

How can the theory of confirmation by reinforcement be extended to constructive languages (of general expressiveness)?

A solution will be presented under one single condition:

- the language will be constituted of units (formulae or rules).

The ***pure reinforcement*** $\rho\rho\,(r)$ of a rule r of a theory T wrt. a given evidence $E = \{e_1, e_2, \ldots, e_n\}$ is defined as:

$$\rho\rho\,(r) = \Sigma_{i=1..n}\, card(Proof_r(e_i,T))$$

The ***(normalised) reinforcement*** is defined as:

$$\rho\,(r) = 1 - 2^{-\rho\rho\,(r)}$$

2. NEW MEASURES.

2.3. Measure of Constructive Reinforcement.

The ***mean reinforcement*** $m\rho(T)$ is defined as:

$m\rho\,(T) = \Sigma_{r\in T}\,\rho\,(r)/m$, with m being the number of rules.

PROBLEM: The use of the mean reinforcement measure suffers the appearance of fantastic concepts.

SOLUTION:

The ***course*** $\chi_T(f)$ of a fact f wrt. a theory T is:

$$\chi_T(f) = max_{S\subset Proof(f,\,T)}\,\{\,\Pi_{r\in S}\,\rho\,(r)\,\}$$

2. NEW MEASURES.

2.3. Constructive Reinforcement and Evaluation.

The ***mean course*** $m\chi(T, E)$ of a theory T wrt. an evidence E is defined as:

$$m\chi(T, E) = \Sigma_{e \in E}\, \chi_T(e)/n \qquad \text{with } n = card(E)$$

– Other global values are defined:
  - ***Compensated Mean Course***.
  - ***Consilience***.
  - ***Intensionality***.

– Comparisons between these criteria are established.

– Different extensions are introduced.

2. NEW MEASURES.

2.3. Reinforcement and Inference Processes.

INDUCTION: $m\chi$ is a hypothesis selection criterion.

- It's more informative and robust than the MDL principle.

ABDUCTION: Explanatory facts also reinforce.

ANALOGY: It's shown crucial for increasing reinforcement.

DEDUCTION: $\rho(r)$ is a ***utility*** criterion.

A ***plausibility*** criterion can also be established:

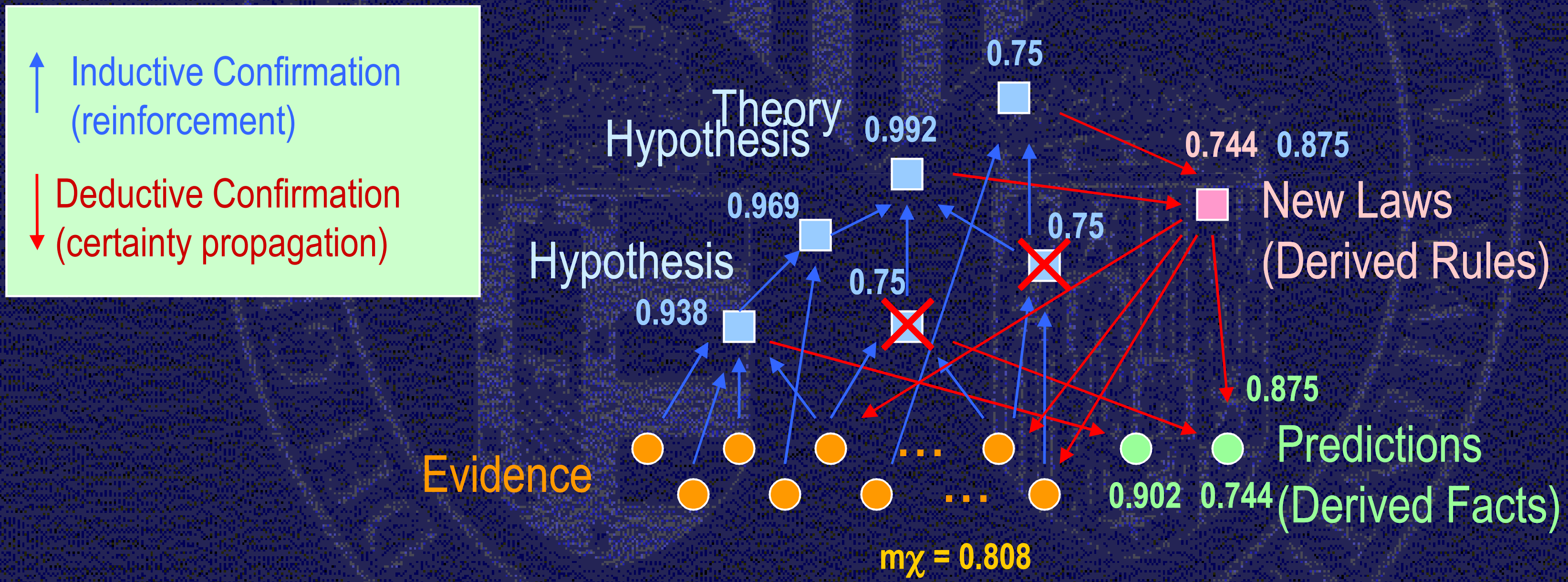
The plausibility of the conclusion is obtained from the reinforcement of the premises:

$$P_1(r) = \chi_T(r)$$

2. NEW MEASURES.

2.3. Reinforcement and Inference Processes.

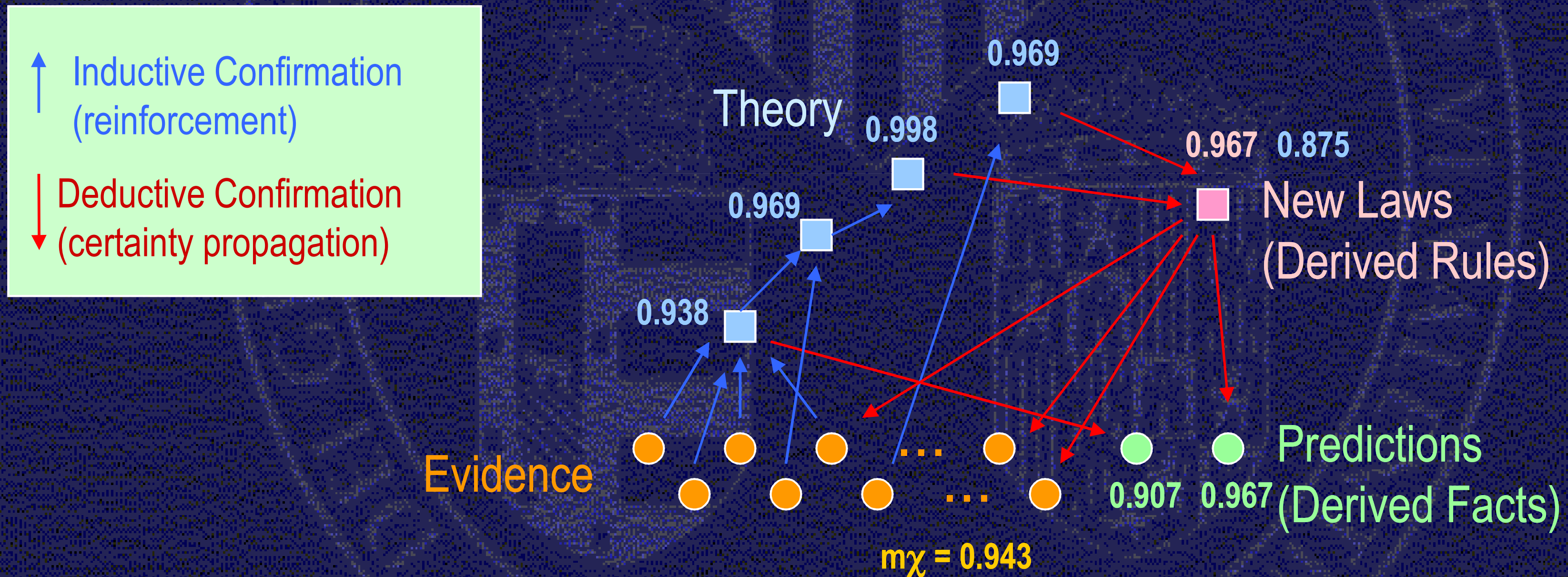
Both inductive and deductive propagation generate reorganisations of a theory.



2. NEW MEASURES.

2.3. Reinforcement and Inference Processes.

Both inductive and deductive propagation generate reorganisations of a theory.



- Not always parts of a theory must be eliminated (forgotten).
- The ***oblivion criterion*** is easily adaptable from $\rho(r)$ and an approximation to G (effort, be it deductive or inductive).

2. NEW MEASURES.

2.3. Characteristics of Constructive Reinforcement.

The measures of reinforcement introduced:

- Valid for constructive languages.
- Adapt consistently all of Hempel's adequacy conditions (sources of confirmation).
- Detailed measure (χ). Gradual and particularised for each constituent of a theory.
- Allow to make predictions with different degree of plausibility (χ_i).
- Allow the construction of different plausibility criteria depending on reinforcement distribution.

2. NEW MEASURES.

2.4. Intensionality and Explanation.

How can an extensional description be formally distinguished from an intensional description (by comprehension)?

Are there intensional descriptions for finite concepts?

A description is intensional (or comprehensive) if it has no *exceptions* to the *pattern* or main rule.

What is an exception?
How can pattern be distinguished?

2. NEW MEASURES.

2.4. Intensionality and Explanation.

- FIRST APPROACH (detection of exceptions): Proportion of the complexity of the general rule wrt. the proportion of the described whole.

  Drawback: It depends on a definition of subprogram.

- SECOND APPROACH (notion of projectability):
  - Notion of projectable description.
  - Notion of equivalence in the limit.
  - Notion of fully projectable description.
  - Notion of stability on the right.

2. NEW MEASURES.

2.4. Intensionality and Explanation.

The *Explanatory Complexity* of an object x given y in a descriptional mechanism β is defined as:

$$Et_\beta(x \mid y) = \min \{ LT_\beta(<p,y>)[..l(x)] - l(y) \text{ such that } <p,y> \text{ is fully projectable} \}$$

We denote with SED($x|y$) the shortest fully projectable description for x given y.

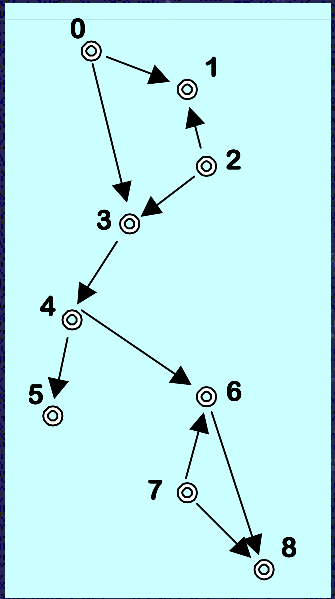
Theorem of Anticipation. There exists a constant c such that for each string x of length n with SED(x) = x^* and $l(x^*) = m$ such that $m < n$, then any split $x = yz$, $l(y) < m - c$ such that SED(y) is not equivalent in the limit with x^*.

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

Conklin & Witten (1994) present an experimental comparison of evaluation criteria:

- the MDL_1 principle based on model complexity.
- the MDL_2 principle based on proof complexity.



Example: (Quinlan 1990) (Conklin & Witten 1994)

- Describes the relation of connection or 'reachability'.
- 9 nodes (0..8).
- The background knowledge *B* is composed of 10 extensional facts of the predicate *linked* :

  B = { linked(0,1), linked(0,3), linked(1,2), linked(3,2), linked(3,4), linked(4,5), linked(4,6), linked(6,8), linked(7,6), linked(7,8) }

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASE 1: Complete Evidence: all the positive examples.

Close World Assumption: the rest is negative.

The evidence *E* is a *complete* specification of the predicate *reach* composed of: 19 facts over 72 possible combinations:

E={ reach(0,1). reach(0,2). reach(0,3). reach(0,4).
reach(0,5). reach(0,6). reach(0,8). reach(1,2).
reach(3,2). reach(3,4). reach(3,5). reach(3,6).
reach(3,8). reach(4,5). reach(4,6). reach(4,8).
reach(6,8). reach(7,6). reach(7,8) }

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 1: **Theories:**

Theory	Program
T_1	reach(X,Y)
T_2	reach(0,1). reach(0,2). reach(0,3). reach(0,4). reach(0,5). reach(0,6). reach(0,8). reach(1,2). reach(3,2). reach(3,4). reach(3,5). reach(3,6). reach(3,8). reach(4,5). reach(4,6). reach(4,8). reach(6,8). reach(7,6). reach(7,8)
T'_2	reach(0,X). reach(3,X). reach(X,8). reach(1,2). reach(4,5). reach(4,6). reach(7,6).
T_3	reach(X,Y) :- linked(X,Y). reach(0,2). reach(0,4). reach(0,5). reach(0,6). reach(0,8). reach(3,5). reach(3,6). reach(3,8). reach(4,8).
T_4	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z). (T'_4)
T_5	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), linked(Z,Y). reach(0,5). reach(0,6). reach(0,8). reach(3,8).
T_6	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), reach (Z,Y).

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 1: **Evaluation :**

T	L(T)	GD	Consilient (without exceps.)	*Gain*	Mean course ($m\chi$)	Spec. ($m'\chi$)	L(E|T)	**MDL$_1$**	PC(E|T)	**MDL$_2$**
T_1	11.5	3.8	Sí	0.57	≈ 1	0.57	56.7	68.2	120.5	**132.0**
T_2	159.5	1	No	0.02	= 0.5	0.5	0	159.5	80.7	240.2
T'_2	60.3	1.52	No	0.59	0.88	0.75	24.3	84.6	100.9	161.2
T_3	111.7	1	No	0.09	0.76	0.76	0	111.7	96.3	208.0
T_4	43.7	2,53	No	0.58	≈ 1	0.67	43.4	87.1	110.6	154.3
T'_4	23.3	2,53	Sí	0.75	≈ 1	0.67	43.4	66.7	123.3	133.9
T_5	94.5	1	No	0.39	0.886	0.89	0	94.5	101.9	196.5
T_6	53.8	1	Sí	0.68	0.999	**0.999**	0	**53.8**	106.1	160.0

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 2: Partial Positive Evidence.

The evidence *E* is now a *partial* specification of predicate *reach* composed of: 12 facts over a total of 19 positive cases.

E={ reach(0,3). reach(0,4). reach(0,5). reach(0,8).
reach(3,2). reach(3,4). reach(3,5). reach(3,8).
reach(4,6). reach(4,8). reach(6,8). reach(7,8) }

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 2: **Theories:**

Theory	Program
T_1	reach(X,Y)
T_2	reach(0,3). reach(0,4). reach(0,5). reach(0,8). reach(3,2). reach(3,4). reach(3,5). reach(3,8). reach(4,6). reach(4,8). reach(6,8). reach(7,8).
T'_2	reach(0,X). reach(3,X). reach(X,8). reach(4,6).
T_3	reach(X,Y) :- linked(X,Y). reach(0,2). reach(0,4). reach(0,5). reach(0,6). reach(0,8). reach(3,5). reach(3,6). reach(3,8). reach(4,8).
T_4	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z). (T'_4)
T_5	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), linked(Z,Y). reach(0,5). reach(0,8). reach(3,8).
T_6	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), reach (Z,Y).

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 2: **Evaluation:**

T	L(T)	GD	Consilient (without exceps.)	Gain	Mean course $(m\chi)$	Spec. $(m'\chi)$	L(E|T)	**MDL$_1$**	PC(E|T)	**MDL$_2$**
T_1	**11.5**	6	Sí	0.57	≈ 1	0.52	43.8	**55.3**	76.1	**87.6**
T_2	101.1	1	No	0.02	= 0.5	0.5	0	101.1	43.0	144.1
T'_2	35.4	2.17	No	≈ 1	0.91	0.66	23.2	58.6	58.9	94.3
T_3	81.9	1.33	No	0.13	0.74	0.66	10.8	92.7	94.1	176.0
T_4	43.7	4	No	0.58	≈ 1	0.56	36.0	79.7	70.9	114.6
T'_4	23.3	4	Sí	0.75	≈ 1	0.56	36.0	59.3	77.9	101.2
T_5	84.5	1.25	No	0.43	0.836	0.77	8.83	93.3	70.3	154.8
T_6	53.8	1.58	Sí	0.68	0.987	**0.82**	15.6	69.4	81.9	135.7

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 3: Partial Positive and Negative Evidence.

The positive evidence E^+ is the same *partial* specification of predicate *reach* composed of: 12 facts over a total of 19 positive cases.

E^+={ reach(0,3). reach(0,4). reach(0,5). reach(0,8).
reach(3,2). reach(3,4). reach(3,5). reach(3,8).
reach(4,6). reach(4,8). reach(6,8). reach(7,8) }

but also:

E^-={ reach(8,3). reach(5,4). reach(0,7). }

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 3: **Theories (the same as case 2):**

Theory	Program
T_1	reach(X,Y)
T_2	reach(0,3). reach(0,4). reach(0,5). reach(0,8). reach(3,2). reach(3,4). reach(3,5). reach(3,8). reach(4,6). reach(4,8). reach(6,8). reach(7,8).
T'_2	reach(0,X). reach(3,X). reach(X,8). reach(4,6).
T_3	reach(X,Y) :- linked(X,Y). reach(0,2). reach(0,4). reach(0,5). reach(0,6). reach(0,8). reach(3,5). reach(3,6). reach(3,8). reach(4,8).
T_4	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z). (T'_4)
T_5	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), linked(Z,Y). reach(0,5). reach(0,8). reach(3,8).
T_6	reach(X,Y) :- linked(X,Y). reach(X,Y) :- linked(X,Z), reach (Z,Y).

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

CASO 3: **Evaluation:**

T	L(T)	GD	Consilient (without exceps.)	Gain	Mean course ($m\chi^0$)	Spec. ($m'\chi^0$)	L(E|T)	**MDL_1**	PC(E|T)	**MDL_2**
T_1	11.5	6	Sí	0.57	0.78	0.50	43.8	**55.3**	76.1	**87.6**
T_2	101.1	1	No	0.02	= 0.5	0.5	0	101.1	43.0	144.1
T'_2	35.4	2.17	No	≈ 1	0.87	0.79	23.2	58.6	58.9	94.3
T_3	81.9	1.33	No	0.13	0.74	0.68	10.8	92.7	94.1	176.0
T_4	43.7	4	No	0.58	0.94	0.63	36.0	79.7	70.9	114.6
T'_4	23.3	4	Sí	0.75	0.94	0.63	36.0	59.3	77.9	101.2
T_5	84.5	1.25	No	0.43	0.836	0.79	8.83	93.3	70.3	154.8
T_6	**53.8**	1.58	Sí	0.68	0.987	**0.86**	15.6	69.4	81.9	135.7

3. APPLICATIONS.

3.1. Evaluation and Generation of Logical Theories.

Reinforcement behaves equal or better than the MDL principle in all of the cases:

- Total positive evidence.
- Partial positive evidence.
- Partial positive and negative evidence.
- Noisy evidence.

El MDL se comporta incluso peor que L(T) en algunos casos.

3. APPLICATIONS.

3.2. Measurement of Intellectual Abilities.

Requirements for evaluating the ability of inference:

- *gradual,*
- *factorial,*
- *non-anthropomorphic,*
- *computationally founded,*
- *meaningful.*

Comprehensibility (Corrected Version). A string x is *k-hard* (or *k-incomprehensible*) given y, denoted by $incomp(x|y)$, in a descriptional system β iff k is the least positive integer number such that:

$$Et_\beta(x|y) \cdot G(SED(x|y) \mid \langle x,y \rangle) \leq k \cdot \log l(x)$$

3. APPLICATIONS.

3.2. Measurement of Intellectual Abilities.

Construction of the ***C-test***:

We choose randomly p sequences $x^{k,p}$, being *k-incomprehensible*, *c-plausible*, *c-m-unquestionable* and *d-stable* with $d \geq r$, with r being the number of redundant symbols of each exercise.

The questions are the K p sequences without their $d - r$ elements ($x^{k,p}{}_{-(d+r)}$). They are given to S and it is asked for the next element according to the best explanation which is able to construct. A fixed time t is given to S and its answers are recorded: $guess(S, x^{k}{}_{-d+r+1})$.

$$I(S) = \sum_{k=1..K} k^{e} \cdot \sum_{i=1..p} hit\left[x^{k,i}_{-d+r+1}, guess(S, x^{k,i}_{-d+r+1})\right]$$

3. APPLICATIONS.

3.3. Other Applications.

Specific Applications:

- Information Systems
- Validation and maintenance of software systems.
- Multi-agent systems, natural language, user interaction, ...

Generic Applications:

- Knowledge acquisition and retrieval.

4. MAIN CONTRIBUTIONS.

- ***A new and more appropriate effective measure of computational information gain*** *G*(*x*|*y*).
- ***New measures of Representation Gain and Representational Optimality***.
- *G*(*x*|*y*) ***is a Uniform Measure for Induction and Deduction***.
- ***A new measure of reinforcement that quantifies the confirmation propagation inside a theory***.
- ***The measure of reinforcement behaves as a measure of confirmation in a consistent way for different inference processes and <u>details</u> the plausibility of rules and predictions***.

4. MAIN CONTRIBUTIONS.

- *The necessity of intermediate information and* ***an oblivion criterion is derived.***
- ***The idea of intensionality is mathematised*** in terms of intolerance or ban of exceptions.
- Definition of an explanatory variant of Kolmogorov complexity as an ***explanatory alternative to the MDL principle.***
- ***A non-anthropomorphic test of intelligence***, based on computational notions and computation theory.
- ***The application of the measures for different kinds of logical systems and knowledge-based systems.***

5. CONCLUSIONS.

The measures and concepts introduced:

- allow a detailed analysis of the value of the output of any inference process wrt. the input and the context, in terms of both informativeness and confirmation.

- have been useful (alone or combined) for formalising, comprehending and relating several relevant notions that have traditionally been rather ambiguous:

  novelty, explicitness/implicitness, informativeness, intensionality, surprise, comprehensibility, consilience, utility, unquestionability, …
- are compatible and can be used to combine and profit the separate advances in the automatisation of different inference processes..