Computational Measures of Information Gain and Reinforcement in Inference Processes

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Reasoning is much more than theorem proving, much more than inductive generalisation and much more than abduction, analogy and other partial inference processes. And it is much more than the sum of all of them. If we can combine consistently and profitably different inference processes, we will enlarge the power and applications of the separate advances in different fields of logic, philosophy of science, artificial intelligence, automated reasoning and machine learning that have taken place in the last half of the XXth century. This would allow that the progress in these different areas would be applied to make intelligent systems, capable of acquiring and deriving new knowledge.

This thesis introduces several evaluation measures which are applicable, in a consistent and effective way, to different inference processes. In particular, these measures are established through two main tools: the theory of Kolmogorov complexity for the definition of explicit information gain and the theory of reinforcement for propagating in a quantitative way the certainty or confirmation degree of deductive and inductive processes. Both tools are not (strictly) semantical, and it is precisely this fact which allows the measurement of different dimensions which have not been tackled successfully to date with purely semantical approaches: informativeness, plausibility, 'consilience', intensionality, intelligibility and utility.

The first part of this work is based on the fact that processes that are apparently so unlike as induction and deduction can be explained in a computational framework as inference processes that both generate an output from an input. Obviously, they must observe different criteria or restrictions, which have been widely studied in philosophy of science and mathematical logic, respectively. In this computational framework, both processes are regarded as non-omniscient processes, i.e., resource-demanding processes. Levin's variant of Kolmogorov Complexity, which weighs the additional amount of information and computational time which is required to solve a problem, is used to define a single gain measure of any inference step or process.

The leading results are obtained by applying the gain measure in an equally clarifying and unifying way to both inductive and deductive processes. In the case of induction, the information gain represents how informative the hypothesis is with respect to the observations, in Popper's sense, and it is compared with other evaluation criteria for induction, mainly simplicity. In the case of deduction, information gain also represents how much informative the conclusion is from the premises, which establishes a generic measure of the gain obtained whenever an explicit knowledge is extracted from an implicit knowledge. In fact, this represents a generalisation of Hintikka's notions of surface and depth information for first-order logic.

Apart from its unifying and explanatory power, the measure of information gain which is presented, although computable, is, as expected, computationally intractable, and it is not directly applicable to concrete systems. Accordingly, a more efficient and detailed measure is introduced, based on the reinforcement or use of the components of an inductive theory or axiomatic system. Reinforcement represents a measure of the confirmation of a theory, which includes the propagation of confirmation by deductive and inductive inference (thus giving a measure of utility or plausibility,

AI Communications
ISSN 0921-7126, IOS Press.

too). Moreover, reinforcement is easy to compute and it is positively related to information gain.

Another connection is established between the idea of implicitness and the notion of intensionality of a description. It is shown not only that extensional definitions have no gain at all but also that intensional definitions, these understood as definitions without exceptions, have a great probability of showing a high information gain. Moreover, the theory of intensionality allows the formalisation of the idea of comprehension, and helps to make the difference between descriptive induction and explanatory induction, the latter requiring that all the evidence should be 'conciliated' by the theory, by avoiding extensional patches.

The previous measures are particularised for logical theories and are compared with other measures in the literature, especially the Minimum Description Length (MDL) principle. It is shown that the measure of reinforcement is more detailed and comprehensive. Furthermore, it solves the problems of induction for finite and random evidences, where the MDL principle suggests the evidence itself and, consequently, nothing is learnt.

The non-omniscient view of inference processes makes it possible to relate the computational capability of a rational agent with several inference problems. More precisely, the difficulty of an instance (or problem) can be defined in terms of the information gain from the problem to the solution and the intrinsic complexity of the solution. A comprehension test is then devised, and correlates, at the sight of results, with classical psychometric tests, representing a formal and non-anthropomorphic alternative to the Turing test.

Finally, several applications of information gain and reinforcement are shown for other inference processes such as abduction or analogy, and many others are sketched for artificial intelligence and computer science: rational agents with limited resources, knowledge-based systems, and knowledge discovery in databases.

In a very succint way, the main contributions of this thesis are:

1. A measure of time-ignoring information gain $V(x|y)$ which represents the degree of information of x which is *implicitly* in y. A new effective measure of computational information gain $G(x|y)$ which measures the proportion of x which is *explicitly* of y.
2. Representation gain, representational optimality measures and a general notion of simplification are also defined from computational information gain.
3. A new measure of reinforcement that behaves appropriately as a detailed measure of confirmation for different inference processes such as induction, abduction, analogy and deduction.
4. Gain and reinforcement act as a 'perfect team' to discern which rules should be left explicitly in the representation of a theory.
5. The idea of intensionality is formalised in terms of avoidance of exceptions, these seen as extensional (non-validated) parts.
6. An explanatory variant of Kolmogorov Complexity allows to define an explanatory counterpart to the MDL principle.
7. A non-anthropomorphic test of intelligence, based on computational and information-theoretic notions, can entail an important advance in the evaluation of AI progress.

Although the main measurements, computational information gain, reinforcement and intensionality, are defined independently, they (alone or combined) are useful to formalise and better comprehend different concepts which have been traditionally rather ambiguous: novelty, explicitness / implicitness, informativeness, surprise, interestingness, comprehensibility, consilience, utility, unquestionability, ...

All in all, the most important result of this work is an operative clarification of the relationship between the notions of inference, effort, information and confirmation. As a conclusion, the view of induction and deduction as inverse processes in terms of information gain is definitively dismissed for non-omniscient systems and for agents with limited-resources, human beings and computers included among them.

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The complete dissertation in English is available at `http://www.dsic.upv.es/~jorallo/tesi/`