Inverse Narrowing for the Inductive Inference of Functional Logic Programs[1]

J. Hernandez-Orallo & **M.J. Ramirez-Quintana**

DSIC, Universitat Politècnica de València
Camí de Vera s/n, 46022 València, Spain.
Email: {`jorallo,mramirez`}`@dsic.upv.es`

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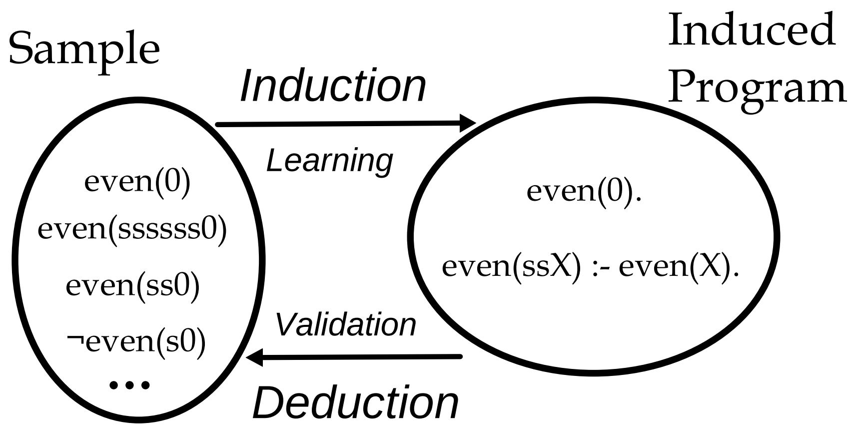
Wide Context

Inductive Synthesis of Declarative Programs



Logic Programming $\cap$ Machine Learning

↘ ↙

Inductive Logic Programming (ILP)

Applications:

- Established: *Scientific Theory Formation, Data Mining, Specific Industrial Applications (Traffic Control)*.
- Promising: *NLP*, *Modelling*, *Program Synthesis.*

Specific Trend

LP Extensions & Combinations

CLP, AILP, Planification (EvC, SitC), RL

Extend ILP to other Declarative Paradigms

- Functional Programming
  Based on rewriting (e.g. Haskell, ML). → Olson95
- Functional Logic Programming
  Based on residuation (e.g. Escher) → FlaGirLlo98
  Based on narrowing (e.g. Curry) → *
- Higher-Order Frameworks
  Different rewriting or unification mechanisms.

Advantages:

- *Background knowledge can be richer: schemata, biases…*
- *More expressive power → More compact theories*
- *The relation between deduction & induction can be more deeply considered (incompleteness, information-gain…)*

Drawbacks:

- *Similar efforts and techniques could be scattered among different representation mechanisms.*
- *In general*, the deduction methods are less efficient or less well-established than resolution.*

Language:

Inductive Functional Logic Programs:

Conditional Rewriting Systems (CRT)
with rules of the form:
$l = r \Leftarrow e_1, \ldots, e_n$ with $n \geq 0$

+

ε-unification

Subsumes LP and Functional Programming.

Narrowing:

- *Sound and complete ε-unification method.*
- *More expressive power in comparison to functional languages.*
- *Better operational behavior in comparison to logic languages.*
- *Migration to HOL will be easier than directly from ILP.*

Narrowing

This work → unconditional case:

- unrestricted (ordinary) narrowing:

Narrowing ($\hookrightarrow$)   *pattern-matching* → *unification*

t 'narrows' into t' ($t \hookrightarrow_\theta t'$) using program P iff

- $u \in O_{nv}(t)$,
- $l = r$ is a new variant of a rule from P,
- $\theta = mgu(t|_u, l)$, and
- $t' = \theta(t[r]_u)$.

Example: program P_1={r_1: X+0=X. r_2: X+sY=s(X+Y) }

$\Leftarrow$ $\underline{\text{s0 + Z}}$ = ss0 u = lhs$|_\varepsilon$, rule r_2, θ={X/s0, Z/sY}

$\Leftarrow$ s($\underline{\text{s0 + Y}}$) = ss0 u = lhs$|_1$, rule r_1, θ={X′/s0, Y/0}

$\Leftarrow$ ss0 = ss0 $X'' = X''$ θ={X′′/ss0}

$\Leftarrow$ *true*   **SOL: { Z/s0 }**

Unrestricted narrowing is sound and complete wrt. canonical programs.
For this work, we shall only induce canonical programs.

Inductive framework

- Evidence E:
  Positive sample E^+
  Negative sample E^-

- Background Knowledge Theory B:

A program P is a solution to the inductive (or learning) problem generated from E iff:

$B \cup P \models E^+$ *(posterior sufficiency or completeness)*
$B \cup P \not\models E^-$ *(posterior satisfiability or consistency)*

Additionally, it is usually supposed
$B \not\models E^+$ *(prior necessity)*
$B \not\models E^-$ *(prior satisfiability)*
Also, to approach abduction in an ILP framework:
$P \not\models E^+$ and only facts can be in P.

Hypotheses Selection

For every E there are infinite many solutions

Criteria for generation and selection:

- The shortest one *(the MDL principle)*
  problems → *Non-computable.*
  → *It can leave extensional parts.*
- The most specific one (Plotkin's *lgg*):
  problem → $P = E^{+}$ *is a solution*.
- The least specific one:
  problem → $P = \mathrm{T} - E^{-}$ *is a solution*.
- The most efficient one:
  problem → $P = E^{+}$ is *usually* the most efficient.
- The most 'coherent' one. No part must be left in an extensional way, i.e., all the data must be produced by the same 'main set of rules'.
  problem → *it must be combined with other criteria to avoid 'fantastic' inductions.*

Example 1

- Background Knowledge Theory B:

  $s(X) < s(Y) = X < Y$

  $0 < s(Y) = true$

  $X < 0 = false$

- Evidence E:

  (E_1^+) $0 + 0 = 0$

  (E_2^+) $s0 + s0 = ss0$

  (E_3^+) $0 + s0 = s0$

  (E_4^+) $\cancel{s(}s0 + s0\cancel{)} = \cancel{s}ss0$

  (E_5^+) $\cancel{s(}ss0 + s0\cancel{)} = \cancel{s}sss0$

  (E_1^-) $s0 + 0 = 0$

  (E_2^-) $0 + 0 = s0$

  (E_3^-) $s0 + s0 = s0$

  (E_4^-) $s0 + 0 = ss0$

  (E_5^-) $\cancel{s(0 + 0) = ss0}$

  (E_6^-) $ss0 + s0 = 0$

  (E_7^-) $s0 = 0$

- Possible solutions:

$P_1 = E+$ *Very specific*

$P_2 = \{X+0 = X, 0+X = X, sX+s0 = ssX\}$

$P_3 = \{X+0 = X, X+sY = s(X+Y)\}$ *Short and Coherent*

$P_4 = \{X+0 = X, X+s0 = sX\}$ *Short*

$P_5 = \{X+Y = X \Leftarrow Y = 0, X+sY = sX \Leftarrow Y = 0)\}$

$P_6 = \{X+0 = X, X+Y = Y+X \Leftarrow X<Y, sX+sY = ss(X+Y)\}$ *Coherent*

$P_7 = \{X+0 = X, 0+X = X, sX+sY = ss(X+Y)\}$ *Efficient*

$P_8 = \{X+0 = X, 0+X = X, sX+sY = s(X+sY)\}$

…

General heuristics

No unified criterion for all the applications.

<u>There is no such thing as "the right hypothesis"</u>

The stop-criteria should be parametrised.

$\Downarrow$

The search is guided by an optimality factor weighting some selected criteria.

$$Opt(P) = \alpha \cdot LenF(P) + \beta \cdot CovF^{+}(P) + \gamma \cdot ConF(P) + \delta \cdot \ldots$$

👍Advantages:

- *The same generic algorithm can be used for different applications.*
- *Any information about the supposed 'true' hypothesis can help to select the different criteria and speed up the search.*

👎Drawbacks:

- *The search cannot be fully optimised (it is difficult to prune if the search heuristics are variable)*

- *Hard-completeness results are difficult.*

Used Criteria

$$\boxed{Opt(P) = LenF(P) + CovF^{+}(P) + ConF(P)}$$

LenF = syntactical length of rhs.

Different 'weight': 1 : constants and functors

0.5 : variables

Example: Weight({ ssX + sX → s(ssX + 0) }) = 5.5

LenF(*P*) = $-\sum_{e \in P} \log_2 \mathrm{Weight}(e)$

CovF⁺(*P*) = $\mathrm{card}(e \in E^{+} : P \models e)/\mathrm{card}(E^{+})$

It allows approximate learning.

ConF(*P*) = 1 if *P* has only an equation, otherwise

ConF(*P*) = $1 - \max(\mathrm{card}(e \in E^{+} : P_i \subset P \wedge P_i \models e))/\mathrm{card}(E^{+})$

Example:

$P_1 = \{ r_1, r_2, r_3 \}$ Suppose $\{ r_3 \}$ covers e_5

and $\{ r_1, r_2 \}$ covers e_1, e_2, e_3, e_4

$\mathrm{ConF}(P_1) = 1/5$ → e_5 is clearly an exception.

Different Stop Criteria for different applications:

- *If CovF+ = 1 and ConF > dc (desired consilience) → Appropiate for **program synthesis** (perfect data and coherent programs)*
- *If dc = 0 and CovF+ = 1 the criterion ≈ MDL principle → No information at all about the source.*
- *If dc = 0.5 and CovF+ = 0.8, learning a consilient theory in the presence of errors (with known error ratio = 0.2).*

Main mechanisms

Inverse of matching/substitution → *generalisation*
Inverse of narrowing → "*inverse narrowing*"

Def. 1. Restricted Generalisation (RG)

Given an equation e ≡ { $t = s$ }, *the equation* t′ = s′ *is a restricted generalisation of* e *iff it is a generalisation, i.e.*

$$\exists \theta : t'\theta = t \wedge s'\theta = \mathrm{s}$$

and it does not include fresh variables in the rhs.

$$\forall x\ (x \in Var(s') \Rightarrow x \in Var(t'))$$

Def. 2. Consistent Restricted Generalisation (CRG)

The equation e = { $l_1 = r_1$ } *is a CRG* w.r.t. E^+ *and* E^- *and the theory* $T = B \cup P$ *iff e is a RG for some equation of* E^+ *and there does not exist a narrowing chain* ($s \hookrightarrow^*_{T \cup e} t$) *such that:*

$s{=}t \in E^-$. *(consistency wrt. E^-)*

Example: (*following Example 1*)

Clause { X′ + 0 = X′ } is a CRG of E^+_1
Clause { X + s0 = sX } is a CRG of E^+_2, E^+_3, (E^+_4), E^+_5

Inverse Narrowing

Def.3 Inverse Narrowing ($\hookleftarrow$)

t 'conversely narrows' into t' ($t \hookleftarrow_{\theta} t'$) iff

- $u \in O(t)$,
- $l = r$ is a new variant of a rule from P ,
- $\theta = mgu(t_{|u}, r)$, and
- $t' = \theta(t[l]_u)$.

Reversed Narrowing + CRG = Inverse Narrowing.

Example:
From the equation e_a ={X + s0 = $\underline{\text{sX}}$} select t= sX
We find a new variant {X′ + 0 = $\underline{\text{X}'}$} from P.
Two occurrences: $u_1 = 1$ *gives* t'_1= s(X + 0)
$u_2 = \varepsilon$ *gives* t'_2= sX + 0
giving two equations
$e_{a,1}$ ={X + s0 = $\underline{\text{s(X+0)}}$}
$e_{a,2}$ ={X + s0 = $\underline{\text{sX+0}}$}
It is obvious that both narrow into e_a using P.
The same holds after CRG: $e'_{a,1}$={X + sY = $\underline{\text{s(X+Y)}}$}

Non-incremental Algorithm

Two main sets:

EH: *Set of equations, generated from all CRG of* E^+.

$PH \subset \wp(EH)$ *: set of programs constructed from EH.*

Initially, $PH = \{ \{e\} : e \in EH \}$

Programs are *merged* using inverse narrowing *followed by a CRG*.

On each iteration, until all the data are 'consiliated':

- *The two most optimal programs are selected, provided they cover most of the examples, and they have not been merged before.*
- *Inverse narrowing is made between all the possible occurrences using one equation of each program.*
- *The resulting programs which are consistent and canonical are added to PH. If not, they can be split.*

Several parameters: ***min, step, inarcomb*** are introduced to *temporarily* prune the search tree.

Condition for using *B*: some example does not have any program which covers it with good optimality.

Example (non-incremental)

- Evidence E:

(E_1^+) append([1,2],[3]) = [1,2,3]
(E_2^+) append([c],[a])=[c,a]
(E_3^+) append([],[4])=[4]
(E_4^+) append([a,b],[])=[a,b]
(E_5^+) append([a,b,c],[d,e])=[a,b,c,d,e]

(E_1^-) append([3],[4])=[4,3]
(E_2^-) append([1,2],[])=[1]
(E_3^-) append([1,2,3],[4])=[1,2,3,4,5]
(E_4^-) append([],[a,b])=[b,a]

- From each example, two (min=2) CRG's are generated with the best optimality:

CRG(E_1^+) = {e_1: append(.(X,.(Y,[])), Z) = .(X,.(Y,Z)),
e_2: append(.(X, .(Y,Z)),.(W,Z)) = .(X,.(Y,.(W,Z))) }
CRG(E_2^+) = {e_3: append(.(X,[]),Y)=.(X,Y),
e_4: append(.(X,Y),.(Z,Y)) = .(X,.(Z,Y)) }
CRG(E_3^+) = {e_5: append([],X)=X,
e_6: append(X,.(Y.X)) = .(Y,X) }
CRG(E_4^+) = {e_7: append(X,[])=X,
e_8: append(.(X,.(Y,Z)),Z)=.(X,.(Y,Z)) }
CRG(E_5^+) = {e_9: append(.(Y,.(Z,.(W,V))),X)=.(Y,.(Z,.(W,X))),
e_{10}: append(.(Y,.(Z,.(W,[]))),X)=.(Y,.(Z,.(W,X))) }

Constructed EH and PH, the best solution is {e_1, e_3, e_5, e_9} covering E^+ (with dreadful optimality and no consilience at all).

Example (cont)

- 1st Iteration. 1st Inverse Narrowing Combination.

There is no pair of programs covering 5 or 4 examples. Thus, from those programs covering 3 examples, the most optimal ones are:

P_1 = { append(X,.(Y,[])),Z) = .(X,.(Y,Z)) } covering E_1^+, E_4^+
P_2 = { append([],X) = X } covering E_3^+

giving 3 consistent programs:

P_a = { append(.(X,.(Y,W)), Z) = .(append(W,X),.(Y,Z)),
append([],X)=X) }
P_b = { append(.(X,.(Y,W)), Z) = .(X,.(append(W,Y),Z)),
append([],X)=X }
P_c = { append(.(X,.(Y,W)), Z) = .(X,.(Y,append(W,Z))),
append([],X)=X }

Added to *PH*. The best solution is the same as before.

Example (cont)

- 2^{nd} Iteration. 2^{nd} Inverse Narrowing Combination.

Now, we find two programs covering 4 examples:

$P'_1 = P_a$ = { $e_{1,1}$: append(.(X,.(Y,W)), Z) = .(append(W,X),.(Y,Z)),
$e_{1,2}$: $\underline{\text{append([],X)}}$=X) } covering E_1^+, E_3^+, E_4^+
P'_2 = { $e_{2,1}$: append(.(X,[]), Y) = .(X,$\underline{\text{Y}}$) } covering E_2^+

Select the two rules with the highest optimality: $e_{1,2}$ and $e_{2,1}$. After inverse narrowing and CRG, most of them are inconsistent. After 'splitting', only one of them results consistent and confluent ($e_{1,1}$ is removed):

P_d = {append(.(X,Z),Y) = .(X, append(Z,Y)),
append([], X) = X }
which covers E^+ and has good optimality.

Best Solution: P_d with consilience > 0.5, the stop criterion.

The example shows that if optimality is not used heuristically, the method is not feasible in practice.

Incremental Algorithm

More interactive (the user can stop the sample).

For each new example which is being presented:

- *If it is a positive example: E^+_n,* check for every program $P_i \in PH$:
  1. HIT ($P_i \models E^+_n$): Just recompute the optimalities.
  2. NOT COVERED ($P_i \not\models E^+_n \wedge \text{lhs}(E^+_\text{n})$ is $\downarrow$): = HIT
  3. ANOMALY: Remove all non confluent and inconsistent P_i from PH and prune EH.

  and we generate all the CRG's of E^+_n in EH and extend PH with all the new unary programs.
- *If it is a negative example: E^-_n, we check the consistency for every program $P_i \in PH$ and we act as in either the HIT or as in the ANOMALY cases.*

<u>In any case</u>, the iteration can be 'reactivated' until the best solution complies with the stop-criterion (*or an iteration limit is exhausted*).

The consilience criterion avoids extensional 'patches' for the NOT-COVERED case.

Example (incremental & BK)

Induce the power function from the product function:

B = {0×X=0, sX×Y=X×Y+Y, X+0=X, X+sY=s(X+Y)}
BF = { × } // Only use × and the functors which appear in E.

Example of 9 steps of an interactive session:

1. The first example E_1^+ = {ss0 ↑ ss0 = ssss0} is processed. The first EH could be enormous and must be pruned.
2. The second example E_1^- = {ss0 ↑ sss0 = ssssssss0} does not make any program inconsistent.
3. The third example E_2^+ = {sss0 ↑ ss0 = sssssssss0} is a NOT COVERED case and generates new equations, like {X ↑ Y = ssssssX} or {sX ↑ X = sssssssX}.
   Poor optimality → inverse narrowing between { E_2^+} and B.
   Program P_a = { X ↑ ss0 = X × X } is generated covering all E^+ and with good optimality over other solutions.
   It is offered to the user. The user deems it to be too hasty.
4. Example E_2^- = {sss0 ↑ sss0 = sssssssss0} prunes some programs but P_a is still the best solution.

5. Example E_3^+ = {sss0 ↑ s0 = sss0} is NOT COVERED by all programs. New CRG's are generated like { X ↑ s0 = X } and {ssX ↑ X = ssX }. Until some limit of iterations, the algorithm stops because it does not find a consilient program. The best one is P_5 = { X ↑ s0 = X, X ↑ ss0 = X × X}.
6. Example E_3^-= {ss0 ↑ sss0 = ssss0 } eliminates some uninteresting programs.
7. Example E_4^+ = { 0 ↑ sss0 = 0} is NOT COVERED by all programs. New CRG's are generated like { 0 ↑ X = 0 }.
8. Example E_4^- = {sss0 ↑ ss0 = ssss0} eliminates some uniteresting programs.
9. Example E_5^+ = {ss0 ↑ 0 = s0} is NOT COVERED by all programs. New CRG's are generated like { X ↑ 0 = s0 } or { ss0 ↑ 0 = s0}. The first one is combined with P_5 which contained { X ↑ ss0 = X × X }. This gives equations like {X ↑ sY = (X ↑ Y) × X}, {X ↑ sY = X × (X ↑ Y)} and {X ↑ sY = (X ↑ X) × Y}. Some new programs are constructed using them. One has very good optimality → the algorithm offers it to the user…

Solution guessed at step 9:

{ X ↑ sY = (X ↑ Y) × X
X ↑ s0 = s0 }

The user now considers it's time to stop.

Obviously, any future example can be NOT COVERED or even can make it inconsistent.

Conclusions and Future Work

General framework for the induction of functional logic programs.

✸Two basic operators are introduced:
- *Consistent Restricted Generalisation*
- *Inverse Narrowing*

✸The selection criterion is parametrisable.

✸Adaptation to the ***incremental*** case is immediate due to the notion of consilience (a good solution is sought earlier than the MDL principle suggests).

Current work:
- *conditional extension: based on balanced reinforcement to avoid exceptions as conditions.*
- *comparison with other ILP systems.*

Future work:
- *theoretical results on 'completeness' and complexity.*
- *study of different narrowing techniques (especially needed narrowing) to possibly integrate with Curry.*
- *higher-order logic.*